A NOTE ON THE POLE ASSIGNMENT OF q-D LINEAR SYSTEMS

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ABSTRACT

The method in Chu [1985], for the solution of the pole assignment problem of separable 2-D linear discrete systems with state feedback, is improved and extended to (ii) q-D linear discrete systems, and

(iii) systems with output feedback.

1. INTRODUCTION.

The 2-D linear discrete system, the work of Roesser [1975], has been investigated by various authors recently. (See the references and the literature therein.) For separable systems, solutions of the pole assignment problem were given in Kaczorek [1983, 1985], Kaczorek and Kurek [1984], and Mertzios [1984]. A solution was given by the author in Chu [1986] involving the selection of eigenvectors from various subspaces, and the problem will be solvable if such subspaces are non-trivial. The philosophy was in line with that in Kautsky et al [1985] for 1-D systems. Some results for q-D ($q > 2$) systems can be found in Kaczorek [1985] and Kaczorek and Kurek [1984].

In this note, the result in Chu [1986] is improved and extended to q-D systems. The possibility of solving the output feedback pole assignment problem is also discussed.

2. q-D SYSTEMS

Consider the q-D linear discrete system

$$Ex = Ax + Bu \tag{1}$$

where $x = [x^{(1)}(\nu)^T, \dots, x^{(q)}(\nu)^T]^T$, $\nu = (i_1, \dots, i_q)$;

with $x^{(k)}(\nu) \in \mathbb{R}^{n_k}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$,

and $n = \sum_k n_k$.

Here E denotes the increment operator, with

$$Ex = [x^{(1)}(\nu + e_1)^T , \dots , x^{(q)}(\nu + e_q)^T]^T$$

and e_k is the k-th row of the identity matrix I_q .

The matrices A and B are partitioned and

$$A = (A_{ij} ; i,j = 1, \dots , q) , \quad B = (B_1^T , \dots , B_q^T)^T .$$

The submatrices A_{ij} and B_i are $n_i \times n_j$ and $n_i \times m$ respectively.

Apply the state or output feedback control law

$$u = F x \tag{2a}$$

or

$$u = K C x , \tag{2b}$$

with $F = (F_1 , \dots , F_q)$ and $C = (C_1 , \dots , C_q)$,

yields the close-loop system matrix

$$A_c = (A_{ij} + B_i F_j ; i,j = 1, \dots , q) \tag{3a}$$

or

$$A_c = (A_{ij} + B_i K C_j ; i,j = 1, \dots ,q) . \tag{3b}$$

Denote the partitioning in A_c conformally to that of A by

$$A_c = (\tilde{A}_{ij} ; i,j = 1, \dots , q) .$$

For separable systems (c.f. Kaczorek [1983, 1985], Kaczorek and Kurek [1984], Mertzios [1984], Chu [1986] one has

$$A_{ij} = 0 , i > j . \tag{4}$$

The pole assignment problem is then reduced to finding F or K such that the close-loop system matrix A_c satisfying the separability condition similar to (4), i.e.

$$\tilde{A}_{ij} = 0 \quad , \quad i > j \; , \tag{5}$$

with $\tilde{A}_{ii}$ assigning the desired poles .

An equivalent theory can also be developed with $i > j$ in (4) and (5) replaced by $i < j$.

3. STATE FEEDBACK PROBLEM.

For state feedback problems, similar to Chu [1986], condition (5) will be satisfied if one chooses F_i in the following manner:

$$F_i = B_i^+ (X_i \Lambda_i X_i^{-1} - A_{ii}) + (I_{n_i} - B_i^+ B_i) Z_i \tag{6}$$

for $i = 1, \ldots, q$; with $(.)^+$ denoting the (1.2.3.4) - or Penrose-pseudo-inverse,

$$\Lambda_i = \text{diag} (\lambda_{i1}, \ldots, \lambda_{i,n_i}) \; ,$$

$$X_i = (x_{i1}, \ldots, x_{i,n_i}) \; , \quad \text{and} \quad Z_i = (z_{i1}, \ldots, z_{i,n_i}) \; .$$

The eigenvectors x_{ik} are chosen from

$$x_{ik} \in S_{ik} \cap T_{ik} \; , \tag{7}$$

with $$S_{ik} = \text{Null} \{(I_{n_i} - B_i B_i^+)(\lambda_{ik} - A_{ii})\},$$

$$V_{ik} = \text{Null} \begin{pmatrix} B_{i+1} B_i^+ (\lambda_{ik} - A_{ii}) \\ \cdots \quad \cdots \\ B_{n_i} B_i^+ (\lambda_{ik} - A_{ii}) \\ \hline B_{i+1} (I_{n_i} - B_i^+ B_i) \\ \cdots \quad \cdots \\ B_{n_i} (I_{n_i} - B_i^+ B_i) \end{pmatrix} = \text{Span} \begin{pmatrix} P_{ik} \\ \hline Q_{ik} \end{pmatrix} \tag{8}$$

and $\quad T_{ik} = \text{span}\ (P_{ik}) \quad . \quad (V_{nk} = \mathbb{R}^{n_k}.)$

If $\quad S_{ik} \cap T_{ik} \neq \{0\} \quad$ and $\quad x_{ik} = P_{ik}\, t_{ik}$,

then $\quad z_{ik}$ is chosen to be

$$z_{ik} = Q_{ik}\, t_{ik} \quad . \tag{9}$$

In Chu [1986], Z_i in (6) is chosen to be zero and

and $\quad S_{ik} = V_{ik} = \text{Null} \begin{bmatrix} B_{i+1}\, B_i^{+}\, (\lambda_{ik} - A_{ii}) \\ \\ B_{n_i}\, B_i^{+}\, (\lambda_{ik} - A_{ii}) \end{bmatrix}$,

not incorrectly but unnecessarily restrictively failing to exploit the column null spaces of B_i . In the modified version in this section, the subspaces T_{ik} are larger and thus $S_{ik} \cap T_{ik}$ have more chances to be nontrivial.

Theorem 1 in Chu [1986] can then be rewritten as follows for q-dimensional systems:-

THEOREM 1. For separable q-D systems,

if

(a) $(A_{kk}\ ,\ B_k)$, $k = 1, \ldots, q$; are completely controllable,

(b) $S_{ik} \cap T_{ik} \neq \{0\}$;

(c) X_i , with $x_{ik} \in S_{ik} \cap T_{ik}$, are non singular;

then $F = (F, , \ldots, F_q)$, with F_i chosen as in (6)-(9), will solve the pole assignment problem with poles $\{\lambda_{ik}\}$.

4. OUTPUT FEEDBACK PROBLEMS

Let us assume that $q = 2$ for simplicity, and assume that

$(A_{ii}\ ,\ B_i\ ,\ C_i)$, $i = 1, 2$; are completely controllable and observable, with rank (B_i) + rank $(C_i) \geq n_i$. (c.f. Chu and Nichols [1985],)

Let X_i^{-1} in (6) be denoted by $Y_i^H = (y_{i1}, \ldots, y_{i,n_i})^H$,

with $(.)^H$ denoting the Hermitian.

From Chu and Nichols [1985], Kautsky et al [1985] and the references therein, it is easy to prove that the eigenvectors y_{ik} can be chosen from

$$y_{ik} \in W_{ik} = \text{Null}_{\{(I_{n_i} - C_i^+ C_i) \,.\, (\bar{\lambda}_{ik} - A_{ii})\}} \quad , \tag{10}$$

similar to the definition of S_{ik} for x_{ik} in (7) .

The feedback gain matrix K in (3b) can then be chosen to be

$$K_1 = B_1^+ (X_1 \Lambda_1 Y_1^H - A_{11}) C_1^+ + (I - B_1^+ B_1) Z_1 + Z_2 (I - C_1 C_1^+)$$

$$+ (I - B_1^+ B_1) Z_3 (I - C_1 C_1^+) \tag{11a}$$

or $$K_2 = B_2^+ (X_2 \Lambda_2 Y_2^H - A_{22}) C_2^+ + (I - B_2^+ B_2) Z_4 + Z_5 (I - C_2 C_2^+)$$

$$+ (I - B_2^+ B_2) Z_6 (I - C_2 C_2^+) \quad , \tag{11b}$$

with the assumption that

$$Z_2 (I - C_1 C_1^+) = 0 \quad \text{and} \quad (I - B_2^+ B_2) Z_5 = 0 \quad , \tag{12}$$

otherwise part of Z_2 or Z_5 can go into Z_3 or Z_6 respectively .

Condition (5) is then equivalent to, for 2-D systems, from (11) ,

$$B_2 K C_1 = 0$$

$$\Longleftrightarrow \quad B_2 B_1^+ (X_1 \Lambda_1 - A_{11} X_1) + B_2 (I - B_1^+ B_1) Z_2 C_1 X_1 = 0 \quad , \tag{13a}$$

or $$(\Lambda_2 Y_2^H - Y_2^H A_{22}) C_2^+ C_1 + Y_2^H B_2 Z_5 (I - C_2 C_2^+) C_1 = 0 \quad , \tag{13b}$$

using the fact that

$$(I - B_2 B_2^+) (X_2 \Lambda_2 - A_{22} X_2) = 0$$

and $$(\Lambda_1 Y_1^H - Y_1^H A_{11})(I - C_1^+ C_1) = 0 ,$$

which are the definitions of W_{ik} .

From (13), further restriction on x_{ik} and y_{ik} can then be deduced, and we have to have

$$\begin{bmatrix} B_2 B_1^+ (\lambda_{ik} - A_{11}) , & B_2(I - B_1^+ B_1) \end{bmatrix} \begin{bmatrix} x_{ik} \\ g_{ik} \end{bmatrix} = 0 \qquad (14a)$$

and

$$(y_{2k}^T , h_{2k}^T) \begin{bmatrix} (\lambda_{2k} - A_{22}) C_2^+ C_1 \\ (I - C_2 C_2^+) C_1 \end{bmatrix} = 0 . \qquad (14b)$$

Matrices Z_2 and Z_5 can then be chosen to be

$$Z_2 = (g_{11} , \ldots , g_{1,n_1}) . Y_1^H . C_1^+ \qquad (15a)$$

and

$$Z_5 = B_2^+ . X_2 . (h_{11}, \ldots , h_{1,n_2})^H \qquad (15b)$$

because of (12) and (13).

The other Z_i's (apart from $i = 2,5$) can then be chosen in (11) to make sure that

$$K_1 = K_2 . \qquad (16)$$

Summarizing the above discussion, we can then solve the output feedback pole assignment problem for 2-D separable systems, if

(a) $(A_{ii} , B_{ii} C_i)$ are completely controllable and observable, with rank (B_i) + rank $(C_i) \geq n_i$.

(b) S_{ik} and the subspace defined in (13a) for x_{ik} has a non-trivial intersection.

(c) W_{ik} and the subspace defined in (13b) for y_{ik} has a non-trivial intersection.

(d) X_i , Y_i are non-singular and

$$Y_i^H X_i = I_{n_i} .$$

(e) Z_1 , Z_3 , Z_4 , Z_6 can be chosen such that K_1 and K_2 in (11) are equal.

Note that the output feedback problem is a difficult one, even for 1-D systems. Appromixate assignment techniques in Chu and Nichols [1985] may have to be used, so that the above restrictions (a) - (e) do not have to be satisfied exactly, with poles $\{\lambda_{ik}\}$ only assigned appromixately. Similar techniques may be applicable to q-D systems, and a lot more work has to be done on output feedback problems.

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